

Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy

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*The views expressed in this presentation are those of the authors and **do not** necessarily represent the views of the Bank of Spain, the ECB, the Eurosystem or the IMF.*

How does firm heterogeneity affect the optimal conduct of monetary policy

- ▶ Firm heterogeneity affects the transmission of monetary policy (e.g. [Ottonello and Winberry, 2020](#); [Jeenas, 2019](#); [Koby and Wolf, 2020](#); [Jungherr et al., 2022](#), ...)
- ▶ One particular channel of interest is through changes in the allocation of capital when financial frictions matter ([Reis 2013](#), [Gopinath et al 2017](#), [Asriyan et al. 2021](#), ...).
- ▶ Which are the implications of firm heterogeneity and financial frictions for the **optimal conduct of monetary policy**?
 - ▶ Challenge: net-worth / productivity distribution is an infinite-dimensional object.

What we do: analyze monetary policy in a model with heterogeneous firms and capital misallocation

- ▶ Benchmark model to understand the [impact of monetary policy on misallocation](#) and endogenous TFP.
 - ▶ Standard New Keynesian block.
 - ▶ Heterogeneous firms block as in [Moll \(2014\)](#).

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 - ▶ Standard New Keynesian block.
 - ▶ Heterogeneous firms block as in [Moll \(2014\)](#).
- ▶ [New algorithm](#) to solve nonlinearly for Ramsey optimal policies with heterogeneous agents using continuous time and Dynare.
 - ▶ Previous methods by [Nuño and Thomas \(2020\)](#), [LeGrand, Martin-Baillon, and Ragot \(2019\)](#) and [Bhandari et al \(2020\)](#)

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 - ★ Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms (“[misallocation channel](#)”)
 - ▶ **Empirical support** for the mechanism based on Spanish firm-level micro data.

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 - ★ Expansionary monetary policy increases investment of more productive firms relatively more, channeling resources towards high-productivity constrained firms (“**misallocation channel**”)
 - ▶ **Empirical support** for the mechanism based on Spanish firm-level micro data.
- ▶ **Optimal monetary policy:**
 - ▶ Misallocation creates a *time inconsistent* motive to temporarily expand the economy.
 - ▶ **Timeless** response to demand shocks: approximate “**divine coincidence**”...
 - ▶ ... but at the **ZLB: low for much longer**.

Literature Review (non-exhaustive!)

- ▶ **Misallocation.** *Hsieh and Klenow (2009, 2014), Restuccia and Rogerson (2013), Moll (2014)...*
 - ▶ Introduce **monetary policy** in a New Keynesian framework.
- ▶ **Monetary policy with firm heterogeneity.** *Ottonello and Winberry (2020), Jeenas (2019), Koby and Wolf (2020), Jungherr et al. (2022)...*
 - ▶ Solve **optimal monetary problem**.
- ▶ **Optimal Monetary Policy with Heterogeneous Agents.** *Acharya, Challe and Dogra (2020), Bhandari et al (2019), Nuño and Thomas (2020), Le Grand, Martin-Baillon, and Ragot (2019)...*
 - ▶ Focus on **heterogeneous firms**.
 - ▶ Simple, general **computational** approach.

Road map

1 Model

2 Positive analysis

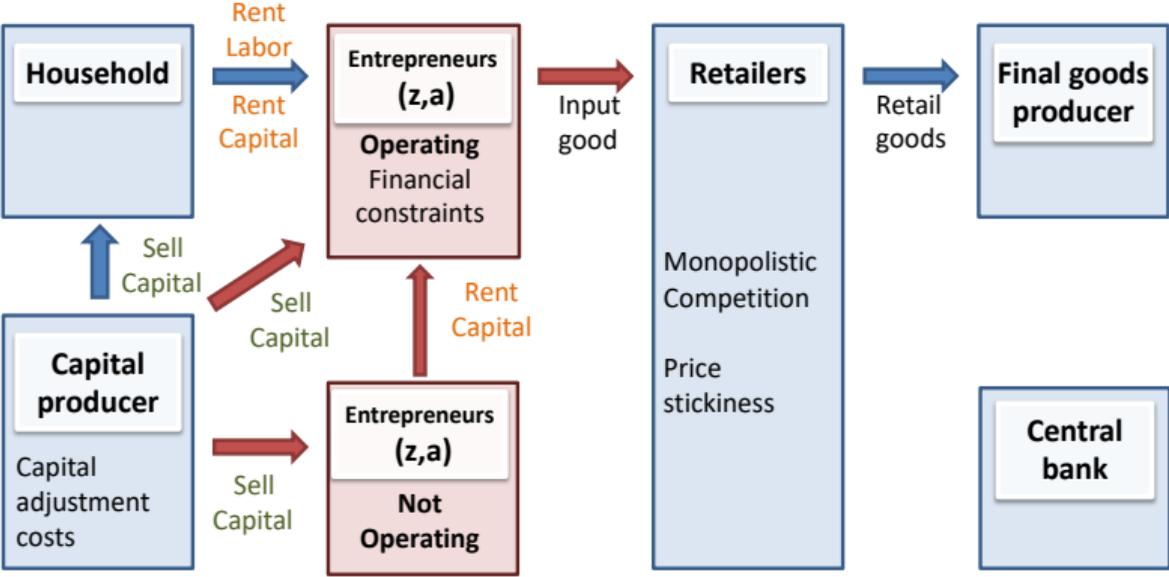
- ▶ Monetary Policy shock
 - ★ Channels
 - ★ Empirical evidence
- ▶ Demand shock

3 Normative analysis

- ▶ Computational methodology
- ▶ Optimal monetary policy
- ▶ Response demand shock (timeless)
- ▶ Response demand shock with ZLB (timeless)

Model

The model in a nutshell



Continuum of heterogeneous firms operated by entrepreneurs

- ▶ Heterogeneity in entrepreneurs' net worth (a_t) and productivity (follows OU-diffusion process, $d\log(z_t) = -(1/\theta) \log z dt + \sigma \sqrt{1/\theta} dW$);
- ▶ Firms produce the input good using labor (l_t) and capital (k_t) with CRS technology.
- ▶ Entrepreneurs can borrow capital $b_t = k_t - a_t$, subject to a borrowing constraint $k_t \leq \gamma a_t$.

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- ▶ Firms maximize profits:

$$\begin{aligned}\Phi_t(z_t, a_t) &= \max_{k_t, l_t} \{ m_t f_t(z_t, k_t, l_t) - w_t l_t - R_t k_t \} \\ \text{s.t. } k_t &\leq \gamma a_t\end{aligned}$$

- ▶ m_t : real price of input good p_t^y / P_t
- ▶ $f_t(z_t, k_t, l_t) \equiv (z_t k_t)^\alpha (l_t)^{1-\alpha}$
- ▶ w_t : real wage

- ▶ R_t : real rental rate of capital
- ▶ $\gamma > 1$: borrowing constraint

Entrepreneurs' optimal production plan with CRS

$$k_t(z, a) = \begin{cases} \gamma a, & \text{if } z \geq z_t^*, \\ 0, & \text{if } z < z_t^*, \end{cases}$$

$$z_t^* = \frac{R_t}{\alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}}$$

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 - ▶ She **lends her net worth** to other entrepreneurs.

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- ▶ If $z \geq z_t^*$, operate at maximum capacity $k_t(z, a) = \gamma a \rightarrow$ Entrepreneur is **constrained**
- ▶ **Optimal choices and profits** are linear in capital

$$\Phi_t(z, a) = \underbrace{\left(z_t \quad \overbrace{\varphi_t}^{\text{Average MRPK}} \quad -R_t \right)}_{\tilde{\Phi}_t(z)} \underbrace{\gamma a}_{k_t}.$$

Entrepreneurs maximize the discounted flow of dividends

- ▶ Entrepreneurs can pay dividends d_t or accumulate net worth a_t ; they retire at rate η .
- ▶ Entrepreneurs are household's members (as in [Gertler & Karadi, 2011](#), unlike [Moll, 2014](#)).

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$$V_0(z, a) = \max_{a_t, d_t \geq 0} \mathbb{E}_0 \int_0^{\infty} e^{-\int_0^t (r_s + \eta) ds} \left(d_t + \overbrace{\eta q_t a_t}^{\text{liquidation value}} \right) dt$$

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s.t.

$$\dot{a}_t q_t + d_t = \underbrace{\left(\overbrace{\max\{\tilde{\Phi}_t(z), 0\} \gamma}^{\text{operating profits}} + \overbrace{\left(\frac{R_t - \delta q_t}{q_t} \right)}^{\text{return on capital}} \right)}_{S_t(z)} q_t a_t$$

▶ Entrepreneurs optimally never distribute dividends until liquidation.

▶ **Intuition:** return of funds inside the firm is always at least the real rate $\left(\frac{R_t - \delta q_t}{q_t}\right)$, and the liquidation value of the firm is all its net worth .

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- ▶ New entrepreneurs enter replacing exiting ones.
 - ▶ Inherit the same firm (same productivity)
 - ▶ Start with lower net worth $\psi q_t a_t$, $0 < \psi < 1$.

Distribution of entrepreneurs

- ▶ The evolution of the **joint distribution** of net worth and productivity $g_t(z, a)$ is given by the KFE:

$$\frac{\partial g_t(z, a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a} [g_t(z, a) s_t(z) a]}_{\text{Entrepreneurs' savings}} \underbrace{-\frac{\partial}{\partial z} [g_t(z, a) \mu(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2} [g_t(z, a) \sigma^2(z)]}_{\text{idiosyncratic TFP shocks}} \\ \underbrace{-\eta g_t(z, a)}_{\text{Entrepreneurs retire}} \underbrace{+\eta g_t(z, a/\psi)/\psi}_{\text{New entrepreneurs}},$$

Distribution in net worth shares and aggregation

- ▶ Entrepreneur's behavior is linear in net worth but nonlinear in productivity.
- ▶ Only need the distribution of **net worth shares** $\omega_t(z) = \frac{1}{A_t} \int_0^\infty ag_t(z, a) da$.

$$\frac{\partial \omega_t(z)}{\partial t} = \left[s_t(z) - \frac{\dot{A}_t}{A_t} - (1 - \psi)\eta \right] \omega_t(z) - \frac{\partial}{\partial z} \mu(z) \omega_t(z) + \frac{1}{2} \frac{\partial^2}{\partial z^2} \sigma^2(z) \omega_t(z)$$

Distribution in net worth shares and aggregation

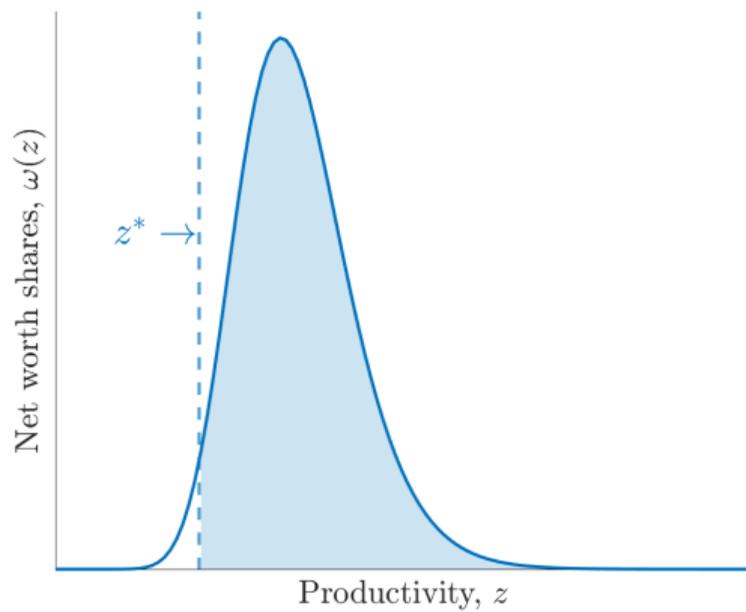
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- ▶ Model is isomorphic to standard RANK with **endogenous** TFP \tilde{Z}_t .
- ▶ Aggregate output Y_t and TFP \tilde{Z}_t are

$$Y_t = \tilde{Z}_t K_t^\alpha L_t^{1-\alpha}, \quad \tilde{Z}_t = \left(\underbrace{\mathbb{E}_{\omega_t(z)} [z \mid z > z_t^*]}_{\text{Endogenous TFP}} \right)^\alpha.$$

Misallocation leads to endogenous TFP



$$\tilde{Z}_t = (\mathbb{E}_{\omega_t(z)} [z \mid z > z_t^*])^\alpha$$

New Keynesian Block

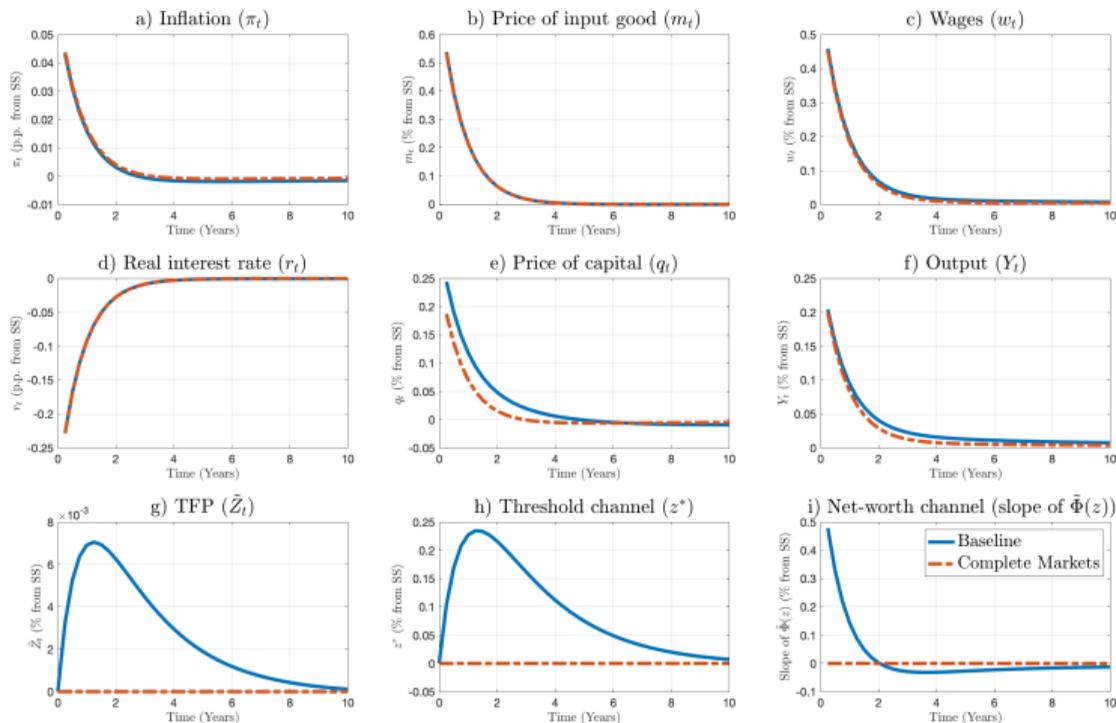
- ▶ Representative Household [▶ More](#)
- ▶ Capital good producer [▶ More](#)
- ▶ Retailers [▶ More](#)
 - ▶ New Keynesian Phillips Curve [▶ More](#)
- ▶ Final good producers [▶ More](#)
- ▶ Central Bank [▶ More](#)

Calibration

	Parameter	Value	Source/target
ρ^h	Household's discount factor	0.025	Av. 10Y bond return of 2.5% (FRED)
δ	Capital depreciation rate	0.065	Aggregate depreciation rate (NIPA)
ψ	Fraction firms' assets at entry	0.1	Av. size at entry 10% (OECD, 2001)
η	Firms' death rate	0.12	Av. real return on equity 11% (S&P500)
γ	Borrowing constraint parameter	1.43	Corporate debt to net worth of 43% (FRED)
α	Capital share in production function	0.3	Standard
ζ	Relative risk aversion parameter HH	1	Log utility in consumption
ϑ	Inverse Frisch Elasticity	1	Kaplan et al. (2018)
Υ	Constant in disutility of labor	0.71	Normalization $L = 1$
ϕ^k	Capital adjustment costs	10	VAR evidence
ϵ	Elasticity of substitution retail goods	10	Mark-up of 11%
θ	Price adjustment costs	100	Slope of PC of 0.1
$\bar{\pi}$	Inflation target	0	-
ϕ	Slope Taylor rule	1.25	-
ν	Persistence Taylor rule	0.8	-
Γ	SS Aggregate Productivity	1	Normalization
ς_z	Mean reverting parameter	0.8	Persistence Gilchrist et al. (2014)
σ_z	Volatility of the shock	0.30	Volatility Gilchrist et al. (2014)

Monetary policy transmission

Expansionary monetary policy shock increases TFP...

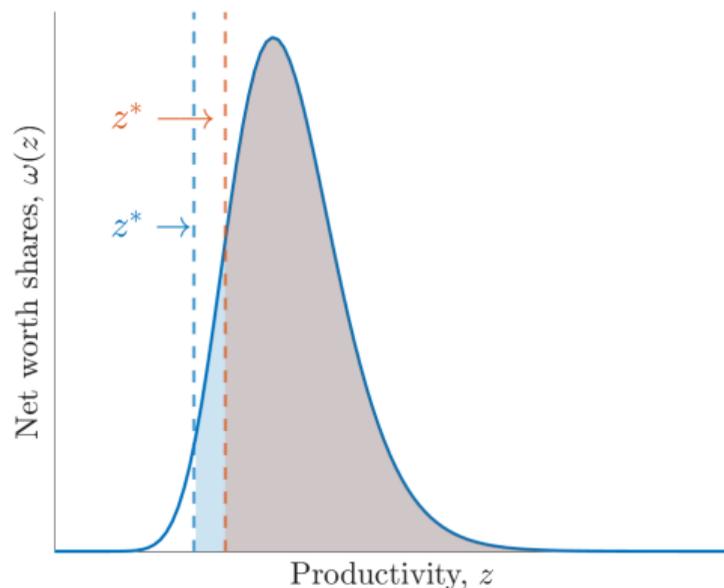


Response to a 25bps decrease in nominal rates

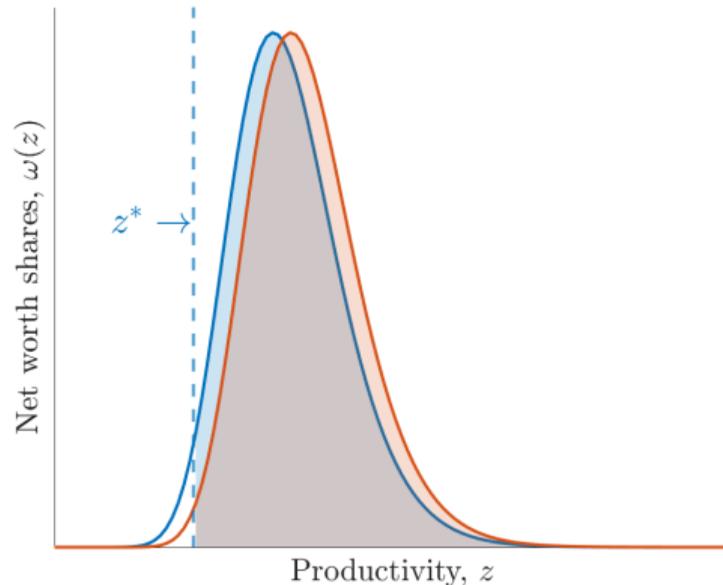
... by reducing misallocation

Monetary policy affects TFP by reducing the share of constrained firms in the economy (z_t^* - *productivity-threshold channel*) and by redistributing resources towards high productivity firms ($\omega_t(z)$ - *net-worth distribution channel*)

Productivity-threshold channel



Net-worth distribution channel



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After a monetary policy expansion, high productivity firms increase their investment relatively more than low productivity firms.

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- ▶ **Data:** yearly balance sheet and cash flow data for the quasi-universe of Spanish firms.
- ▶ **Monetary policy shocks** identified à la Jarociński and Karadi (2020). [▶ more](#)
- ▶ Empirical specification following Ottonello and Winberry (2020):

$$\Delta \log k_{j,t} = \alpha_j + \alpha_{st} + \beta (MRPK_{j,t-1} - \mathbb{E}_j [MRPK_j]) \varepsilon_t^{MP} + \Lambda' Z_{j,t-1} + u_{j,t}.$$

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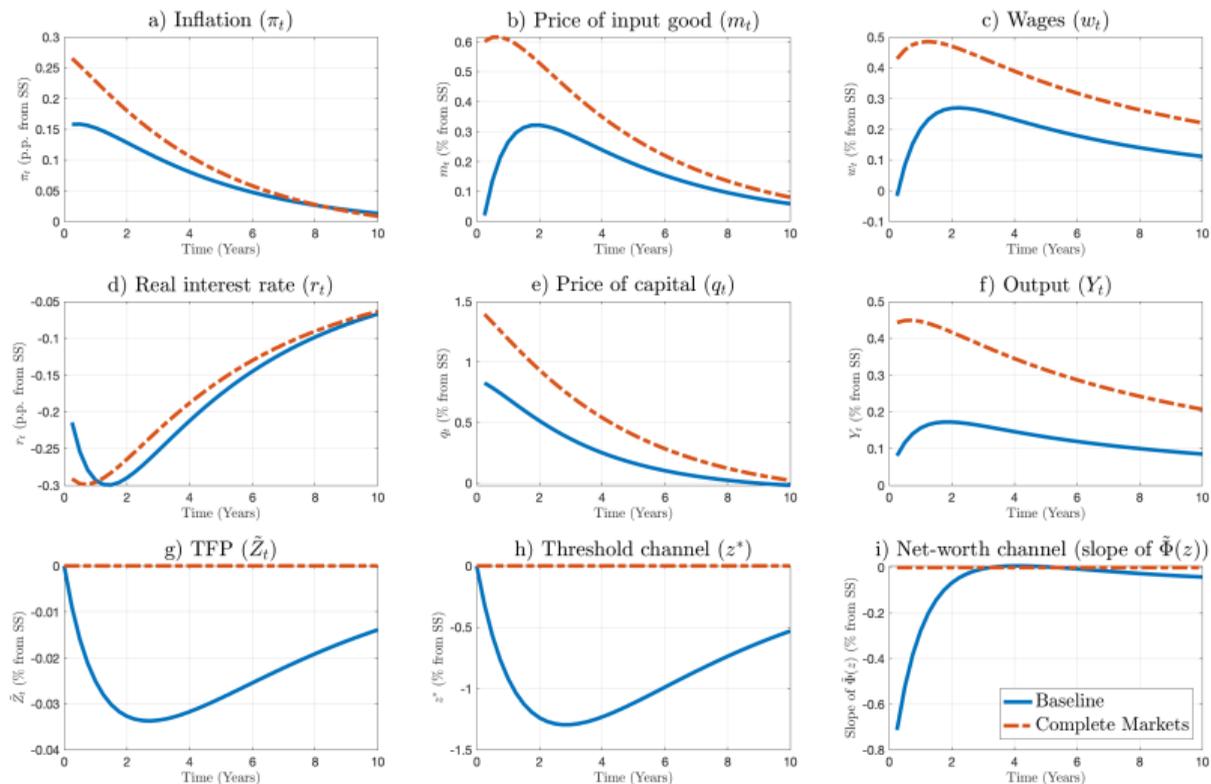
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	(1)	(2)
$\varepsilon_t^{MP1} \times MRPK_{t-1}$	0.141** (0.06)	0.293*** (0.07)
Observations	5,567,706	4,169,950
R^2	0.267	0.285
MRPK control	YES	YES
Controls	NO	YES
Time-sector FE	YES	YES
Time-sector clustering	YES	YES

Yes!

Demand shock decreases endogenous TFP

In line with the mechanism of Reis (2013), Gopinath et al. (2017), Asriyan et al. (2021).



Optimal Monetary Policy

Central Bank's Ramsey problem

$$\max_{\{\omega_t(z), \text{Prices}_t, \text{Quantities}_t\}_{t \in [0, \infty)}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho^h t} u(C_t, L_t) dt$$

subject to private equilibrium conditions $\forall t \in [0, \infty)$ and initial conditions

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- ▶ Need to keep track of the whole distribution of firms $\omega_t(z)$
- ▶ We propose a new algorithm to solve for Ramsey optimal policies with heterogeneous agents.
 - ▶ Discretize the continuous time and continuous-space problem and use standard software (Dynare) to solve non-linearly for the optimal monetary policy in the sequence space.

Sketch of solution algorithm

▶ Back

- 1 **Discretize** the time space (Δt); and the state space (Δz) into J grid points using **finite differences** (Achdou et al, 2017):

▶ system of $2J$ equations and $2J$ unknowns for the HJB and the KFE equation (we don't have a HJB).

$$\left(\begin{array}{l} \frac{1}{\Delta t} (\mathbf{v}^{n+1} - \mathbf{v}^n) + \rho \mathbf{v}^{n+1} = \mathbf{u}^{n+1} + \mathbf{A}^{n+1} \mathbf{v}^{n+1} \\ \frac{\mathbf{g}^{n+1} - \mathbf{g}^n}{\Delta t} = (\mathbf{A}^{n+1})^T \mathbf{g}^{n+1} \end{array} \right)$$

▶ set of X equilibrium conditions (MC, FOCs of representative agents)

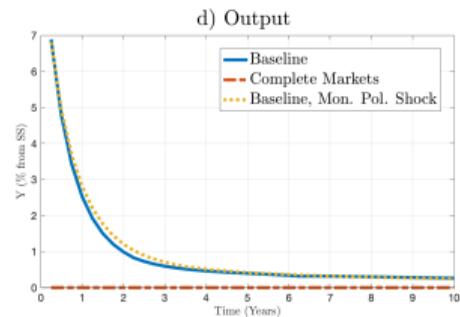
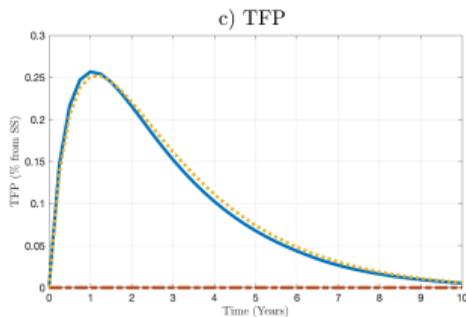
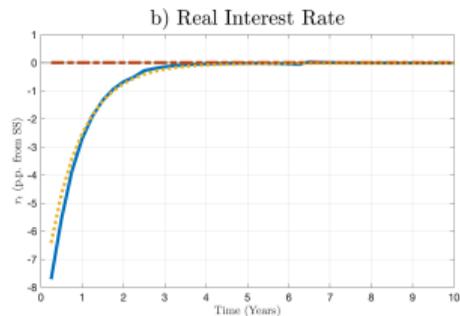
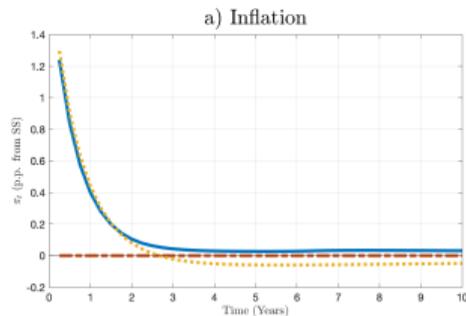
- 2 Compute the **planner's optimality conditions** on discretized problem : $(2J + X) + (2J + X + 1)$ equations using **symbolic differentiation**
- 3 Solve the transitional dynamics up to horizon T using a **Newton algorithm** to solve a large equation set of $[(2J + X) + (2J + X + 1)] T$ equations (cf. Auclert et al., 2020)

▶ Using Dynare

Optimal Ramsey policy: a new time inconsistency

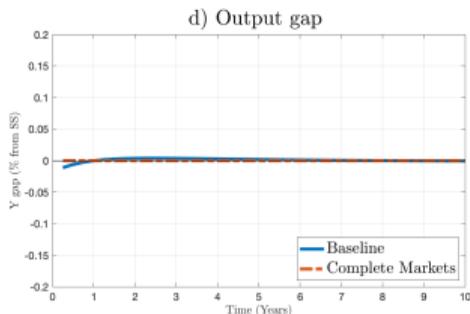
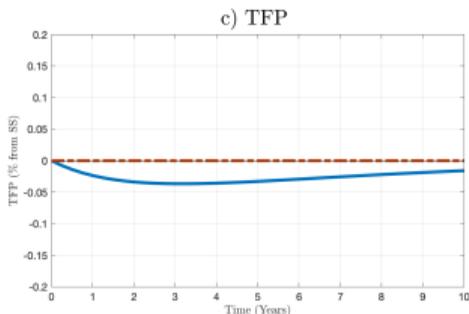
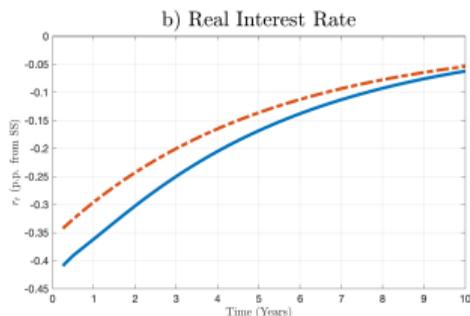
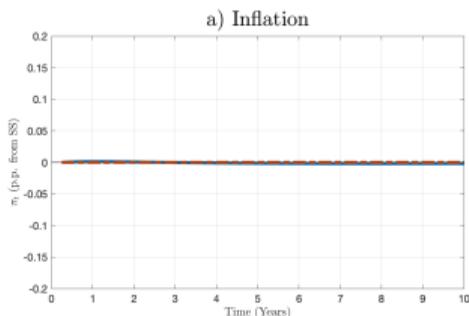
▶ Channels: borrowing constraints

▶ Channels: persistence of idiosyncratic shocks



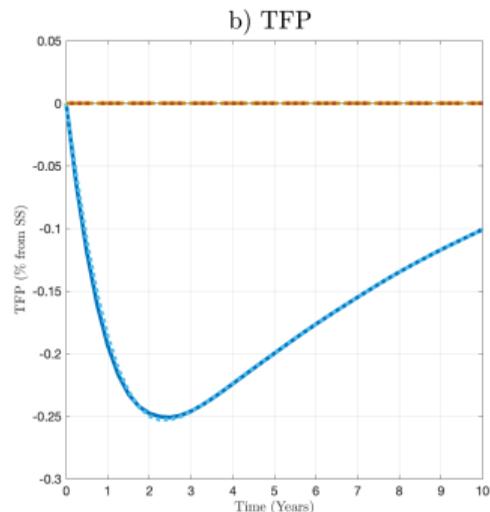
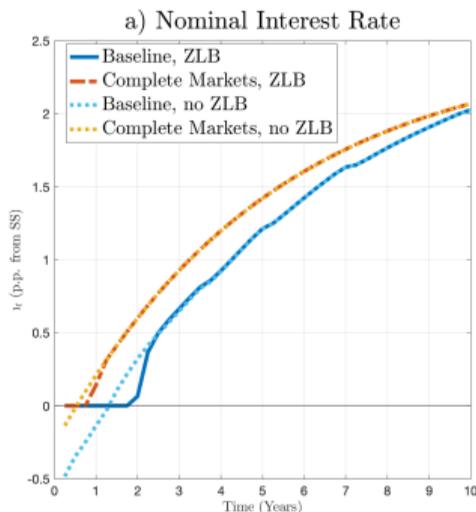
- ▶ Complete Markets economy (CM): **zero inflation** is optimal (steady state is first-best due to subsidy undoing mark-up distortion) ▶ [CE vs Baseline](#)
- ▶ Baseline economy: **surprise inflation** is optimal since it reduces capital misallocation

Timeless optimal response to a demand shock: divine coincidence holds



- ▶ Exogenous decrease in HH subjective discount rate: increase in savings of the HH reduces the return to savings of entrepreneurs, which makes low productivity firms start operating, increasing the share of constrained firms in the economy and shifting away resources from high productivity entrepreneurs.

Timeless optimal response to a demand shock with ZLB: low for even longer



- ▶ If planner were not constrained by ZLB (blue), she would decrease further nominal rates (light blue) as compared to the ZLB case.
- ▶ Heterogeneity and financial frictions calls for '*low for longer*' compared to the complete markets case (orange)

Conclusions

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 - ▶ Empirical evidence supporting higher investment of high productivity firms after expansionary monetary policy shock

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- ▶ **Positive analysis**: expansionary MP reduces misallocation through the productivity-threshold and net-worth channels
 - ▶ Empirical evidence supporting higher investment of high productivity firms after expansionary monetary policy shock
- ▶ **Normative analysis**: important implications for optimal monetary policy
 - ▶ New source of inflationary time inconsistency: undoing financial frictions.
 - ▶ 'Quasi'-divine coincidence holds when facing demand shocks (timeless)
 - ★ Zero-Lower Bound: *Low for even longer.*

Thank you!

Appendix

Representative household

▶ Back

Standard consumption-labor-savings choice

$$\max_{C_t, L_t, D_t, B_t^N} \mathbb{E}_0 \int_0^{\infty} e^{-\rho^h t} u(C_t, L_t) dt$$

s.t.

$$\dot{D}_t q_t + \dot{B}_t^N + C_t = (R_t - \delta q_t) D_t + (i_t - \pi_t) B_t^N + w_t L_t + T_t$$

▶ C_t : consumption

▶ D_t : capital holdings

▶ B_t^N holdings of nominal bonds (zero net supply)

▶ L_t : labor supply

▶ i_t : nominal interest rate

▶ T_t : profits of *retailers*, *capital good producer* and *net dividends* from firms

Capital good producer

Produces capital and sells it to the household and the firms at price q_t

▶ Back

$$\max_{\iota_t, K_t} \mathbb{E}_0 \int_0^{\infty} e^{-\int_0^t r_s ds} (q_t \iota_t - \iota_t - \Xi(\iota_t)) K_t dt.$$
$$\text{s.t. } \underbrace{\dot{K}_t = (\iota_t - \delta) K_t}_{\text{LOM of } K_t}.$$

- ▶ ι_t : investment rate,
- ▶ $\Xi(\iota_t) = \frac{\phi^k}{2} (\iota_t - \delta)^2$: quadratic adjustment costs.

New Keynesian block

▶ Back

- ▶ **Final good producers** aggregate varieties $j \in [0, 1]$. Cost minimization implies demand for variety j is given by

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \text{ where } P_t = \underbrace{\left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}}_{\text{Agg. Price index}}.$$

- ▶ **Retailers** maximize

$$\max_{p_{j,t}} \int_0^{\infty} e^{-\int_0^t r_s ds} \left\{ \underbrace{\left(\frac{p_{j,t}}{P_t} - m_t\right)}_{\text{Mark-up}} \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \frac{\theta}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}}\right)^2 Y_t \right\} dt$$

- ▶ ε : elasticity of substitution across goods

$\varepsilon > 0$.

- ▶ θ : price adjustment cost parameter.

- ▶ $p_{j,t}$: price of variety j .

The

New Keynesian block

▶ Back

- ▶ The symmetric solution to the pricing problem yields the **New Keynesian Phillips curve**

$$\left(r_t - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\varepsilon}{\theta} (m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon},$$

- ▶ $\pi_t = \frac{\dot{P}}{P_t}$ is inflation,
- ▶ m_t are relative prices of intermediate good (inverse mark-ups of retailers),
- ▶ m^* is the optimal inverse mark-up,
- ▶ Real rates are defined as $r_t \equiv \frac{R_t - \delta q_t + \dot{q}_t}{q_t}$.

Central Bank

▶ Back

- ▶ The central bank controls nominal interest rates i_t on nominal bonds held by households.
- ▶ **Positive analysis:** central bank sets the nominal rate according to a Taylor rule of the form

$$di = -v (i_t - (\rho^h + \phi (\pi_t - \bar{\pi}) + \bar{\pi})) dt, \quad (1)$$

where $\bar{\pi}$ is the inflation target, ϕ is the sensitivity to inflation deviations and v is a parameter related to the persistence.

- ▶ **Normative analysis:** we assume that the central bank implements the Ramsey-optimal policy.

▶ Back

$$w_t = (1 - \alpha)m_t Z_t K_t^\alpha L_t^{-\alpha},$$

$$R_t = \alpha m_t Z_t K_t^{\alpha-1} L_t^{1-\alpha} \frac{z_t^*}{\mathbb{E}[z \mid z > z_t^*]},$$

$$\frac{\dot{A}_t}{A_t} = \frac{1}{q_t} \left[\gamma(1 - \Omega(z_t^*)) \left(\alpha m_t Z_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t \right) + R_t - \delta q_t - q_t(1 - \psi)\eta \right].$$

Distribution of entrepreneurs

- ▶ The evolution of the **joint distribution** of net worth and productivity $g_t(z, a)$ is given by the KFE:

$$\frac{\partial g_t(z, a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a} [g_t(z, a) s_t(z) a]}_{\text{Entrepreneurs' savings}} - \underbrace{\frac{\partial}{\partial z} [g_t(z, a) \mu(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2} [g_t(z, a) \sigma^2(z)]}_{\text{idiosyncratic TFP shocks}}$$
$$\underbrace{-\eta g_t(z, a)}_{\text{Entrepreneurs retire}} \quad \underbrace{+\eta g_t(z, a/\psi)/\psi}_{\text{New entrepreneurs}}$$

RANK vs HANK

▶ Back

RANK

- ▶ All capital is owned by HH $D_t = K_t$
- ▶ No financial frictions.
- ▶ TFP is exogenous
 $Z = 1$

HANK

- ▶ Capital is owned by HH and entrepreneurs: $D_t + A_t = K_t$
- ▶ Financial frictions: $k_t \leq \gamma a_t$
- ▶ TFP is endogenous
 $Z = (\mathbb{E}_t [z \mid z > z_t^*])^\alpha$

- ▶ Introduce subsidies in both economies, such that the SS mark-up distortion is undone.

Direct and indirect effects on profits and threshold

▶ Back

Profits

$$\Phi_t(z, a) = \underbrace{(z\varphi_t - R_t)}_{\tilde{\Phi}_t} \gamma a = \left(z\alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}} - (q_t(r_t + \delta) + \dot{q}_t) \right) \gamma a$$

Productivity threshold z^*

$$z_t^* = \frac{R_t}{\varphi_t} = \frac{(q_t(r_t + \delta) + \dot{q}_t)}{\alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}}$$

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▶ Back

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- ▶ $\downarrow q_t$ and $\downarrow r_t$ increase profits *homogenously* for all firms, and *decrease* threshold z^* .

Direct and indirect effects on profits and threshold

▶ Back

Profits

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Productivity threshold z^*

$$z_t^* = \frac{R_t}{\varphi_t} = \frac{(q_t(r_t + \delta) + \dot{q}_t)}{\alpha \left(\frac{(1-\alpha)}{w_t} \right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}}$$

- ▶ $\downarrow q_t$ and $\downarrow r_t$ increase profits *homogenously* for all firms, and *decrease* threshold z^* .
- ▶ $\downarrow w_t$ and $\uparrow m_t$ increase profits *relatively more* for more productive firms, and *decrease* threshold z^* .

Monetary policy transmission

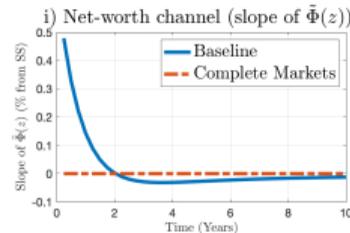
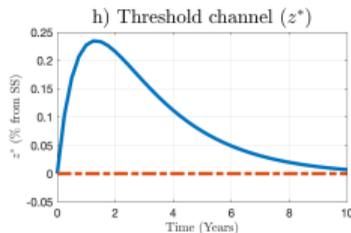
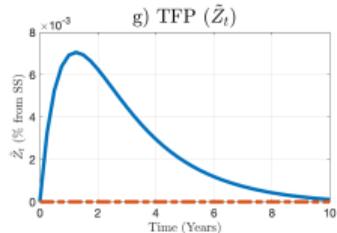
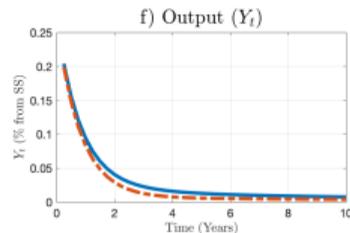
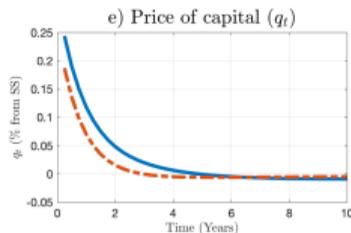
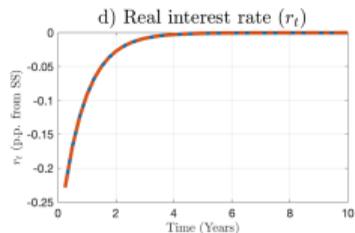
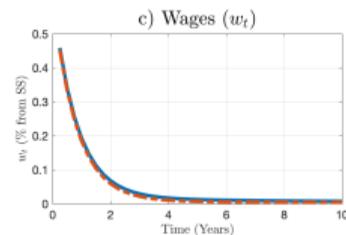
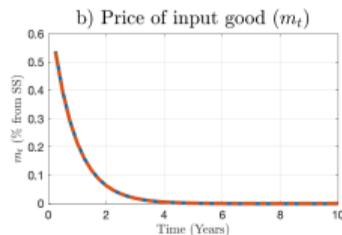
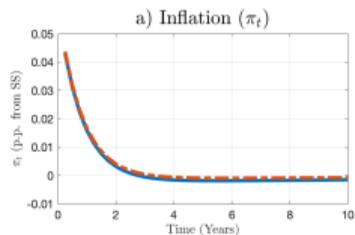
▶ Back

Calibration

	Parameter	Value	Source/target
ρ^h	Household's discount factor	0.025	Av. 10Y bond return of 2.5% (FRED)
δ	Capital depreciation rate	0.065	Aggregate depreciation rate (NIPA)
ψ	Fraction firms' assets at entry	0.1	Av. size at entry 10% (OECD, 2001)
η	Firms' death rate	0.12	Av. real return on equity 11% (S&P500)
γ	Borrowing constraint parameter	1.43	Corporate debt to net worth of 43% (FRED)
α	Capital share in production function	0.3	Standard
ζ	Relative risk aversion parameter HH	1	Log utility in consumption
ϑ	Inverse Frisch Elasticity	1	Kaplan et al. (2018)
Υ	Constant in disutility of labor	0.71	Normalization $L = 1$
ϕ^k	Capital adjustment costs	10	VAR evidence
ϵ	Elasticity of substitution retail goods	10	Mark-up of 11%
θ	Price adjustment costs	100	Slope of PC of 0.1
$\bar{\pi}$	Inflation target	0	-
ϕ	Slope Taylor rule	1.25	-
ν	Persistence Taylor rule	0.8	-
Γ	SS Aggregate Productivity	1	Normalization
ς_z	Mean reverting parameter	0.8	Persistence Gilchrist et al. (2014)
σ_z	Volatility of the shock	0.30	Volatility Gilchrist et al. (2014)

Expansionary monetary policy shock increases TFP...

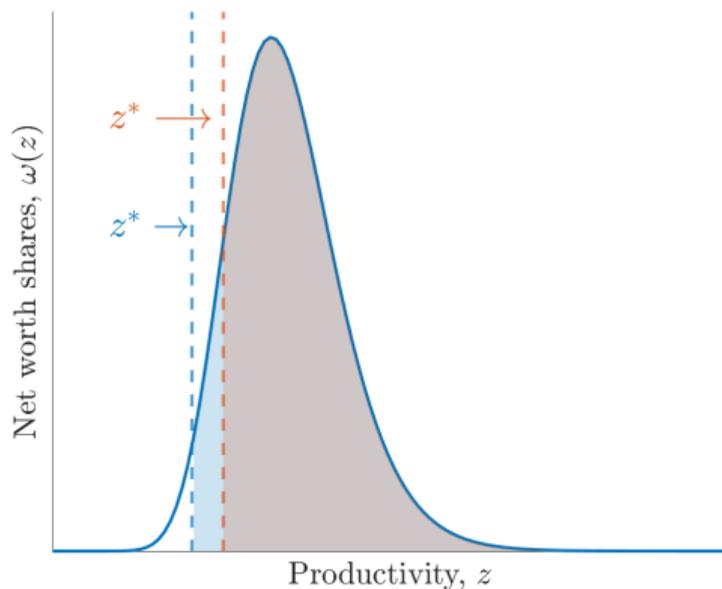
► Demand shock



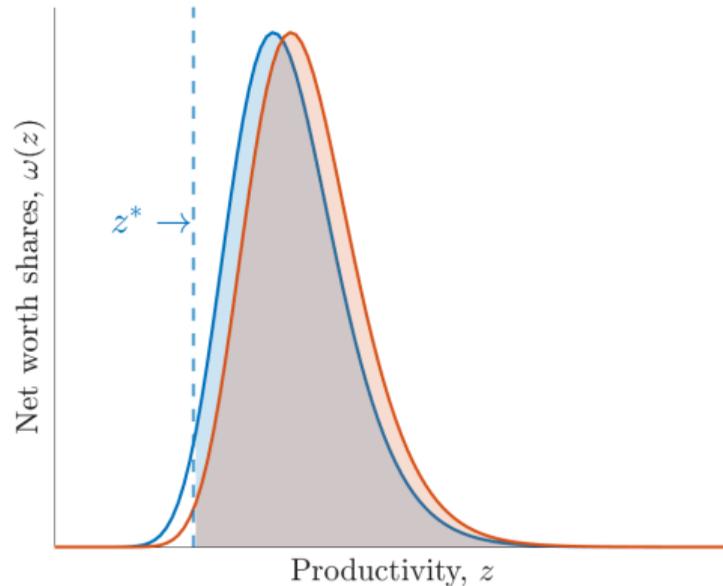
... by reducing misallocation

Monetary policy affects TFP by reducing the share of constrained firms in the economy (z_t^* - *productivity-threshold channel*) and by redistributing resources towards high productivity firms ($\omega_t(z)$ - *net-worth distribution channel*)

Productivity-threshold channel

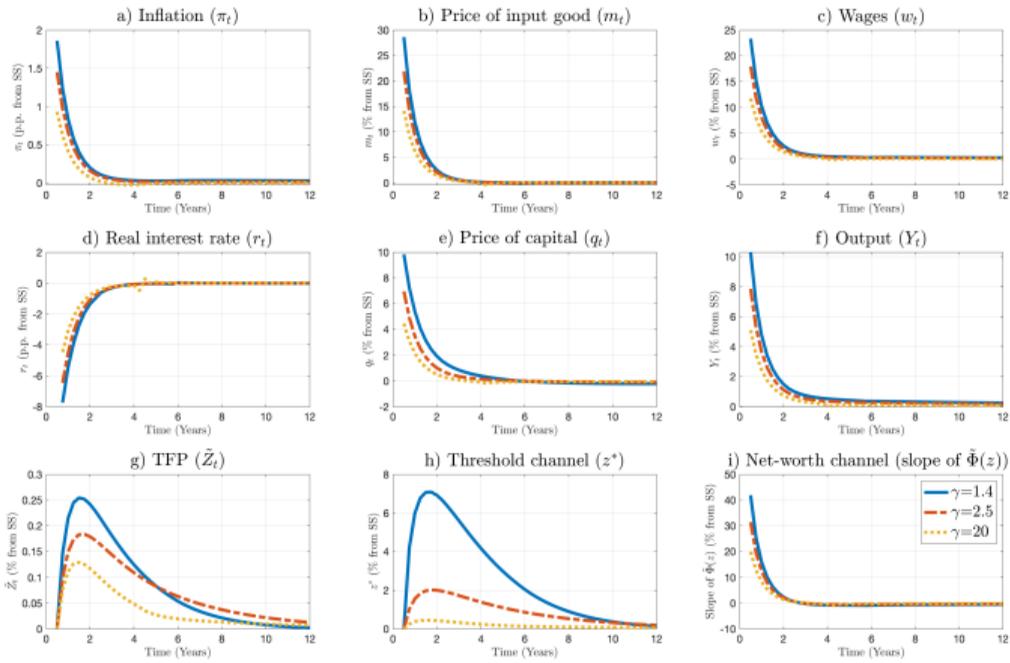


Net-worth distribution channel



The tighter the borrowing constraint is, the more expansionary monetary policy is

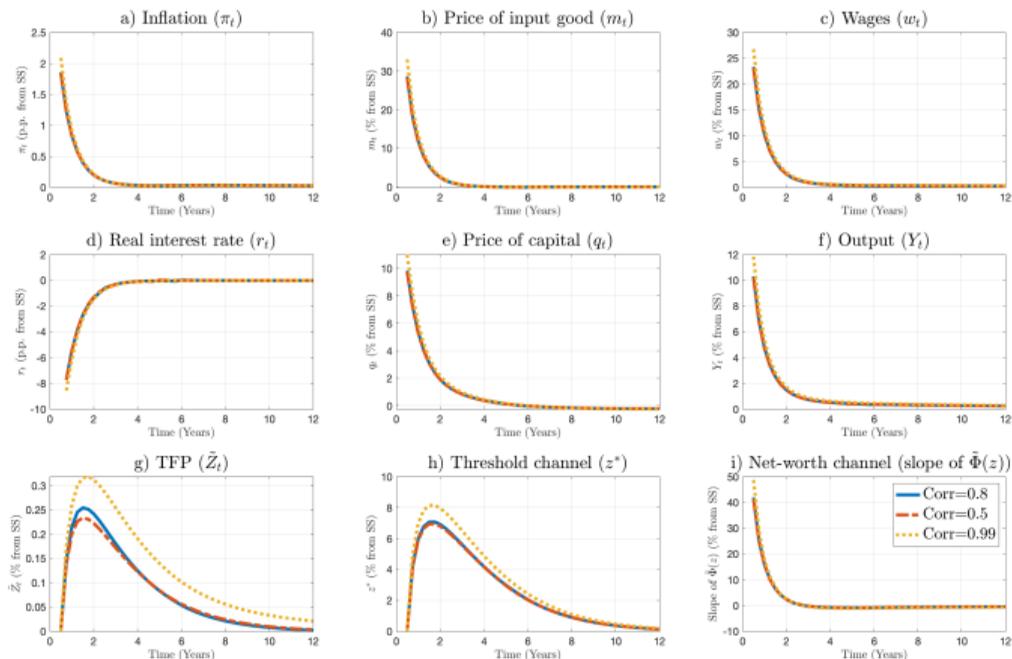
▶ Back



▶ ...since it allows entrepreneurs to undo financial frictions faster.

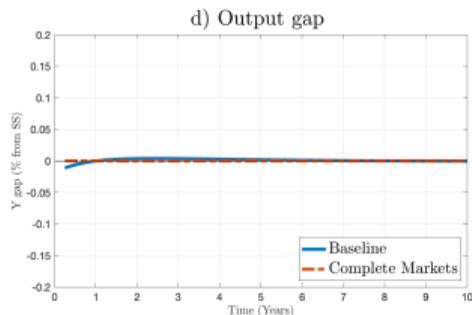
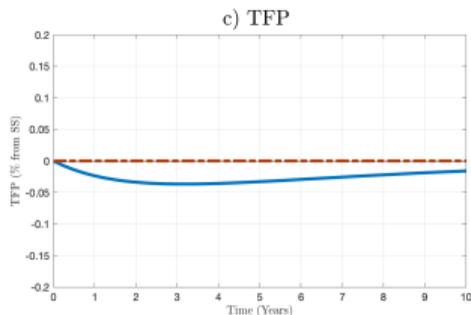
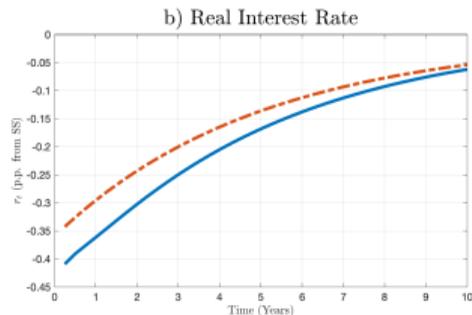
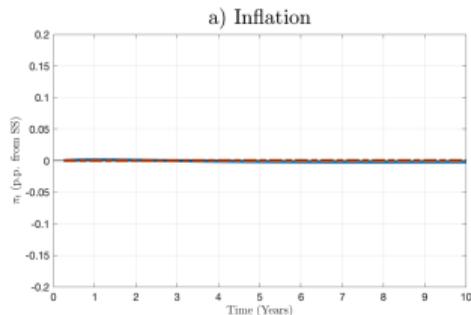
Higher persistence of the idiosyncratic shocks allows the increase in TFP to last longer

▶ Back



- ▶ More productive firms invest relatively more, and since the idiosyncratic shock is persistent, the increase in TFP lasts longer.

Timeless optimal response to a demand shock: 'quasi'-divine coincidence



- ▶ Exogenous decrease in HH subjective discount rate: the increase in savings of HH makes it more expensive for firms to save and undo financial constraints, decreasing TFP.
- ▶ Planner finds a 'quasi'-divine coincidence optimal.

Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

- ▶ Provide
 - ▶ the *SS of the problem* conditional on the policy instrument,
 - ▶ the set of discretized *non-linear equilibrium conditions* of the private economy,
 - ▶ the *planner's objective function*.

Use Dynare to solve the OMP problem in Discrete Time / Discrete Space non-linearly

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- ▶ Use *ramsey_model* command:
 - ▶ Dynare computes FOCs for the Ramsey problem by symbolic differentiation.
- ▶ Use *steady* command:
 - ▶ Dynare computes SS of the Ramsey problem.
- ▶ Use *perfect_foresight_solver* command:
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Easy to use and general!

Can we see this pattern in the data after an expansionary MP shock?

After a monetary policy expansion, high productivity firms increase their investment relatively more than low productivity firms.

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- ▶ **Data:** yearly balance sheet and cash flow data for the quasi-universe of Spanish firms.
- ▶ **Monetary policy shocks** identified à la Jarociński and Karadi (2020). [▶ more](#)
- ▶ Use MRPK as proxy for productivity.
- ▶ Empirical specification following Ottonello and Winberry (2020):

$$\Delta \log k_{j,t} = \alpha_j + \alpha_{st} + \beta (MRPK_{j,t-1} - \mathbb{E}_j [MRPK_j]) \varepsilon_t^{MP} + \Lambda' Z_{j,t-1} + u_{j,t}.$$

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	(1)	(2)
$\varepsilon_t^{MP1} \times MRPK_{t-1}$	0.141** (0.06)	0.293*** (0.07)
Observations	5,567,706	4,169,950
R^2	0.267	0.285
MRPK control	YES	YES
Controls	NO	YES
Time-sector FE	YES	YES
Time-sector clustering	YES	YES

Yes!

Empirical evidence: Details

MP shock

- ▶ high-frequency data and sign restrictions in a SVAR to identify monetary policy shocks in the Euro area at the monthly level, aggregated at a yearly frequency.
- ▶ renormalized so that ε_t^{MP} is a 100bps expansionary monetary policy shock.

Empirical evidence: Details

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Productivity

- ▶ $MRPK_t = \frac{\partial m_t f_t(z, k, l^*)}{\partial k} = \left[\left(\frac{1-\alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}} m_t^{\frac{1}{\alpha}} \right] z \propto z$.
- ▶ Demean MRPK to ensure that the results are not driven by permanent heterogeneity in responsiveness across firms.
- ▶ Controls $Z_{j,t-1}$ include: MRPK, total assets, leverage, sales growth, net financial assets as a share of total assets, MRPK \times GDP growth.

Empirical evidence: Robustness

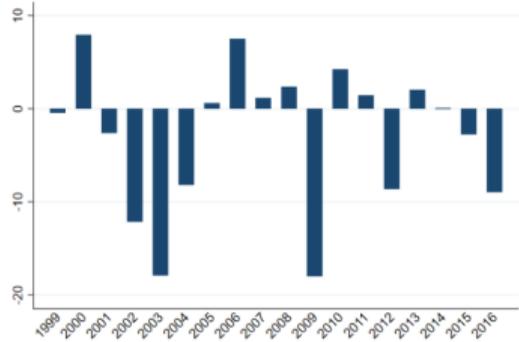
▶ Back

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ϵ_t^{MP} x $MRPK_{t-1}$	0.238*** (0.06)	0.299*** (0.07)	0.177** (0.07)	0.432*** (0.09)				
Inv_{t-1}	-0.0310*** (0.00)	-0.0259*** (0.00)						
ϵ_t^{MP2} x $MRPK_{t-1}$					0.166* (0.10)	0.345*** (0.10)		
ϵ_t^{MP} x $MRPK_{t-1}$ (not demeaned)							0.0906** (0.04)	0.243*** (0.04)
Observations	4,162,114	4,094,537	283,835	263,397	5,567,706	4,169,950	5,567,706	4,169,950
R^2	0.279	0.283	0.153	0.162	0.267	0.285	0.267	0.286
MRPK control	YES	YES	YES	YES	YES	YES	YES	YES
Controls	NO	YES	NO	YES	NO	YES	NO	YES
Time-sector FE	YES	YES	YES	YES	YES	YES	YES	YES
Time-sector clustering	YES	YES	YES	YES	YES	YES	YES	YES
Panel	FULL	FULL	BALANCED	BALANCED	FULL	FULL	FULL	FULL

MP shocks

▶ Back

Panel 1 - Baseline weighting - ϵ_t^{MP}



Panel 2 - Alternative weighting - ϵ_t^{MP2}

